9/2/25 Lecture 3

Proof Example:

$$\neg q \to \neg(\neg p \vee q)$$

$\equiv \neg\neg q \vee \neg(\neg p \vee p)$ conditional property

$\equiv q \vee \neg(\neg p \vee p)$ double negation

$\equiv q \vee \neg(T)$ complement laws

$\equiv q \vee F$ negation ?

$\equiv q$ identity ?

$S(x)$ $x^2 < x$

domain of x: $x \in (0, 1)$

So we say this: $\forall_x S(x)$ or $\forall x \in (0.5, 0.6)\ S(x)$

$S(x)$ $x^2 < x$

domain of x: $x \in (0.9, 1.1)$

so we say this: $\exists_x S(x)$ or $\exists x \in (0.9, 1.1)\ S(x)$

$\mathbb{Z}$ → Set of all integers (positive and negative)

$\mathbb{R}$ → Set of all real numbers

$\mathbb{Q}$ → Set of all rational numbers

$\mathbb{C}$ → set of all complex numbers

$\mathbb{N}$ → set of all natural numbers

Counterexample → any x that disproveds $\forall x\, S(x)$
example → any x that proves $\exists x\, S(x)$

Free variable → can take on any value in the domain.
Bound variable → $\forall x\, P(x)$ is a bound variable because its tied to a quantifier.

Will be Tested in Exam!

$M(x)$: x came to the meeting on time
$O(x)$: x is an officer
$D(x)$: x has paid the does.
$x \in$ All members of club

1. Someone is not an officer
$\exists x \neg O(x)$

2. Everyone came to the meeting
$\forall x\, M(x)$

☆ 3. All the officers came on time to the meeting
$\forall x\, (O(x) \rightarrow M(x))$

☆ 4. There is an officer who did not come on time
$\exists x\, (O(x) \rightarrow \neg M(x))$ Incorrect
$\exists x\, (O(x) \land \neg M(x))$ Correct

Tips for Words to quantifiers (will be on Test)

$\forall x$ usually goes with $\rightarrow$

$\exists x$ usually goes with $\wedge$

$\forall x\ O(x) \rightarrow D(x)$

"Every officer paid for their dues."

$\exists x\ (M(x) \wedge \neg D(x))$

"There exists a member who came on time and did not pay dues."

De Morgans Law

$$\neg \forall x\ P(x) \equiv \exists x\ \neg P(x)$$

$$\neg \exists x\ P(x) \equiv \forall x\ \neg P(x)$$

Truth values of $\forall x P(x)$ and $\exists x P(x)$ when domain empty.

$\forall x P(x)$: T

$\exists x P(x)$: F

FCE

$\neg (\forall x < 0 : x^2 > 0)$

Hint: $\forall x < 0 : x^2 > 0 \equiv \forall x \, (x < 0 \rightarrow x^2 > 0)$

Answer: $\exists x < 0 : x^2 \leq 0$